Mid-term Exam #1

MATH 106 - A&B Winter 2016

Name: ____

Instructions:

- Answer as many of the following questions as possible.
- No cell phones or collaboration allowed. If you leave the classroom during the exam you must leave your cell phone with the instructor.
- Approved calculators are allowed.
- Additional scrap paper is available upon request.
- *Multiple choice questions:* Circle the letter corresponding to your answer. No partial credit will be awarded.
- *Short answer questions:* Show all of your work on the page of the problem. Clearly indicate your answer and the reasoning that you used to arrive at the answer. You do not have to simplify algebraic expressions.

This exam has 5 multiple choice problems and 5 short answer problems. There are a total of 100 points.

Good luck!

Problem	**Possible Points**	**Points Earned**
MC	25	
6	18	
7	18	
8	15	
9	12	
10	12	
TOTAL	**100**	

1. (5 points) Let $I = \int_a^b f(x)\,dx$ and suppose that $f(x)$ is decreasing and concave down on the interval $[a, b]$. Which of the following statements is true?

   A. $L_n \leq I \leq R_n$

   B. $L_n \leq I \leq T_n$

   C. $I \leq L_n \leq R_n$

   D. $T_n \leq I \leq L_n$

   E. $R_n \leq L_n \leq I$

2. (5 points) Imagine a square pyramid with base side s and height h. What is the area of the horizontal cross section at height $h/4$ (measured from the base of the pyramid)?

   A. $\frac{1}{4}s^2$

   B. $s^2 - \frac{1}{4}$

   C. $\frac{9}{16}s^2$

   D. $\frac{3}{4}s^2$

   E. $\frac{1}{16}s^2$

3. (5 points) The following definite integral is performed using substitution. Find a and b:

$$\int_{-1}^{2} \frac{x^2}{\sqrt{x^3 - 2}}\, dx = \frac{1}{3} \int_a^b u^{-1/2}\, du.$$

A. $a = -3, \quad b = 6$

B. $a = -1, \quad b = 2$

C. $a = -1, \quad b = 6$

D. $a = 3x^2, \quad b = 3$

E. $a = 0, \quad b = 4$

4. (5 points) The length of the curve $y = f(x)$ from $x = 0$ to $x = 8$ is given by the integral $\int_0^8 \sqrt{(3x^2 + 2)^2 + 1}\, dx$. Choose the appropriate $f(x)$.

A. $f(x) = 3x^2 + 2$

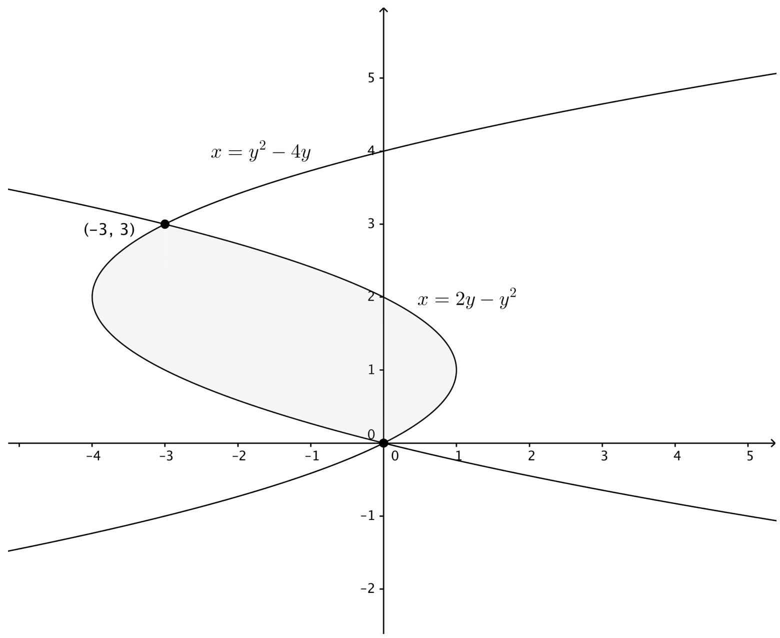
B. $f(x) = 6x$

C. $f(x) = \sqrt{x^3 + 2x + 4}$

D. $f(x) = x^3 + 2x + 4$

E. $f(x) = (3x^2 + 2)^2 + 1$

5. (5 points) Which of the following integrals describes the area of the region depicted below?



A. $\displaystyle\int_{-4}^{1} y^2 - 4y - (2y - y^2)\,dy$

B. $\displaystyle\int_{0}^{3} 2y - y^2 - (y^2 - 4y)\,dy$

C. $\displaystyle\int_{-4}^{1} \sqrt{x + 4y} - \sqrt{2y - x}\,dx$

D. $\displaystyle\int_{0}^{3} y^2 - 4y - 2y - y^2\,dy$

E. $\displaystyle\int_{-4}^{-3} y^2 - 4y\,dy + \int_{-3}^{1} 2y -^2\ dy$

6. (18 points) Compute the following integrals.

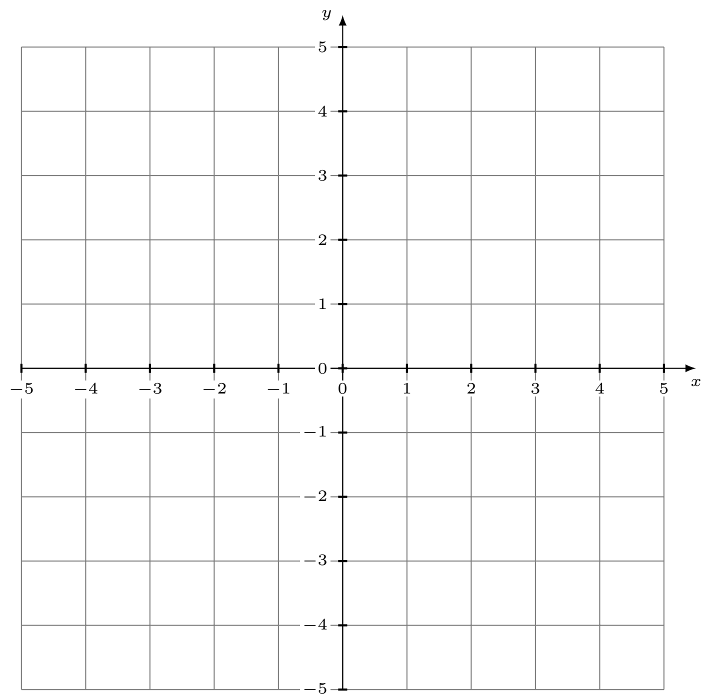
(a) (6 points) $\int x\cos(1+2x^2)\,dx$

(b) (6 points) $\int \frac{x}{(x+4)^2}\,dx$

(c) (6 points) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

7. (18 points)

(a) (4 points) Draw the region enclosed by the curves $y = x$ and $y = 2\sqrt{x}$ and label the points of intersection.



(b) (8 points) Rotate the region in part (a) around the line $y = -3$. The resulting solid of revolution will have vertical cross sections in the shape of a (circle the correct choice):

disk or washer.

The outer radius of the cross section at x is $R =$_______________.

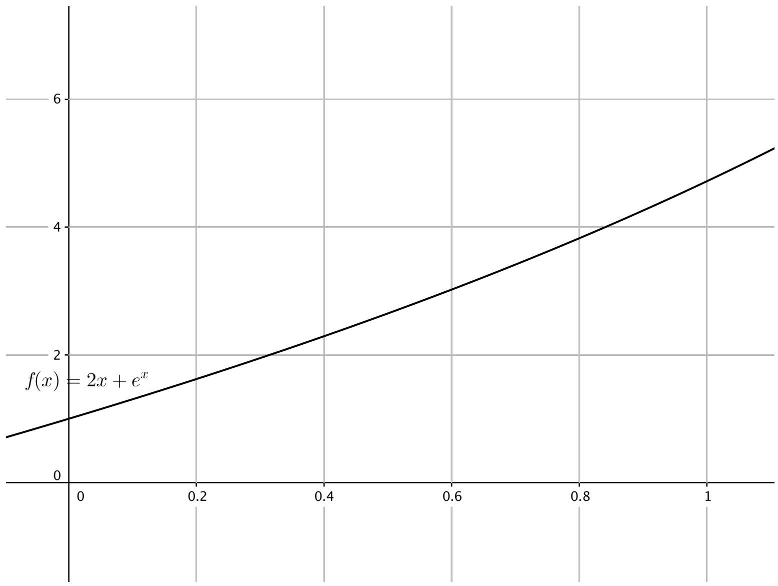
The inner radius of the cross section at x is $r =$_______________.

If the cross section is a disk, fill in only the outer radius above and leave the inner radius blank.

(c) (6 points) Find the volume of the solid obtained by rotating the region in part (a) around the line $y = -3$.

8. (15 points) The identity $\int_0^1 2x + e^x \, dx = e$ gives us a way to approximate the constant e.

(a) (3 points) Draw M_5 on the plot below and compute M_5, the midpoint approximation of e with 5 subdivisions.



$M_5 =$ ______________________

You may use the following theorem to complete the problems on the next page:

Let $I = \int_a^b f(x)\,dx$, *and let* M_n *denote the midpoint sum for* I *with* n *equal subdivisions. Let* K_2 *be a constant such that* $|f''(x)| \leq K_2$ *for all* x *in* $[a, b]$. *Then*

$$|I - M_n| \leq \frac{K_2(b-a)^3}{24n^2}.$$

(b) (6 points) Find an appropriate value for K_2 and then find an upper bound $|e - M_5|$.

(c) (6 points) What is the smallest value of n for which $|e - M_n| < 0.0001$?

9. (12 points) Frank and Jenna have a treehouse that is 12 feet above the ground. They lift their dog Bailey up to the treehouse using a harness and a rope. The combined weight of Bailey and the harness is 75 pounds. The rope weighs 0.5 pounds per foot.

   (a) (4 points) Write the force function.

   (b) (4 points) Frank lifts Bailey from the ground to a height of 4 feet. Find the work done by Frank (your answer will be in ft-lb).

   (c) (4 points) Jenna lifts Bailey from 4 feet above the ground until he reaches the treehouse. Find the work done by Jenna (your answer will be in ft-lb).

10. (12 points) Solve the initial value problem

$$(y^2 + xy^2)y' = 1, \quad y(0) = 1.$$