

Math 106C
Calculus 2
Winter 2016
Exam 1
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Please show your work.

1. a) Draw a curve/graph of a function, $f(x)$, that is increasing and concave up.

b) Based on your sketch in part (a), put the following quantities in **increasing** order.

$$L_{100}, R_{100}, T_{100}, M_{100}, \int_a^b f(x) dx.$$

2. Evaluate by finding an antiderivative. (Your final answer should not contain an integral.)

a) $\int \sqrt{3x-2} dx$

b) $\int \frac{\ln(x)}{x} dx$

3. Evaluate: $\int_1^2 xe^{x^2-1} dx$.

4. A cylindrical tank of radius 3 ft. and height 4 ft. is filled with water. Find the work required to pump all the water over the upper rim. [Note: water weighs 62.4 lbs per cubic foot.]

5. Consider the region bounded by the graph $y = (x - 1)^3 + 1$, $y = x^2$, $x=0$ and $x = 1$.

a) Sketch this region.

b) Find the area of this region.

c) Write (DO NOT evaluate) a definite integral representing the arc-length of the path given by $y = (x - 1)^3 + 1$ from the origin $(0,0)$ to the point $(1,1)$.

6. Find the volume of the solid formed by revolving the area between the functions

$$y = \sqrt{x+1} \text{ and } y = \frac{1}{\sqrt{2}}(x+1) \text{ about the } x\text{-axis. [Start by sketching this region.]}$$

7. Find the solution of the following differential equation:

$$\frac{dy}{dx} = -xy, \text{ where } y(0) = 2.$$

8. Consider a function f given by the following table:

x	0	0.5	1	1.5	2
f(x)	1	4	5	2	-1

a) Find R_4 to estimate $\int_0^2 f(x)dx$.

b) Recall that the error committed by using the Right Hand Sum, R_n , is less than or equal to $\frac{K_1(b-a)^2}{2n}$ where $|f'(x)| \leq K_1$ for some constant K_1 . Suppose that $|f'(x)| \leq 6$ for $0 \leq x \leq 2$. How large do you require n to be in order to guarantee that

$$\left| R_n - \int_0^2 f(x)dx \right| \leq 0.001?$$