**MATH 205A,B - LINEAR ALGEBRA**
**WINTER 2013**

QUIZ 4

**NAME:** **Section:**(Circle one) A(1 : 10) B(2 : 40)

**Show ALL your work CAREFULLY.**

Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by

$$T(x_1, x_2, x_3) = (2x_1 - 3x_2 + x_3, 4x_3 - x_1).$$

(a) Find the standard matrix $A$ of $T$ so that $T(\vec{x}) = A\vec{x}$.

**The matrix $A$ has columns $T(\vec{e_1}), T(\vec{e_2}), T(\vec{e_3})$. Since $T(\vec{e_1}) = T(1,0,0) = (2,-1)$, $T(\vec{e_2}) = T(0,1,0) = (-3,0)$, $T(\vec{e_3}) = T(0,0,1) = (1,4)$, it follows that the matrix $A$ is given by**

$$A = \begin{bmatrix} 2 & -3 & 1 \\ -1 & 0 & 4 \end{bmatrix}.$$

(b) Find all vectors $\vec{x}$ such that $T(\vec{x}) = \vec{0}$.

**Since $T(\vec{x}) = A\vec{x}$, the vectors $\vec{x}$ such that $T(\vec{x}) = \vec{0}$ are precisely the solutions to the homogeneous equation $A\vec{x} = \vec{0}$. Now,**

$$A = \begin{bmatrix} 2 & -3 & 1 \\ -1 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 5 \\ -1 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 5 \\ 0 & -3 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 \\ 0 & -3 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \end{bmatrix}.$$

**The solutions to $A\vec{x} = \vec{0}$ are**

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4x_3 \\ 3x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

**where $x_3$ is the parameter.**

(c) Is $T$ one to one? Explain.

**No. From (b), the homogeneous equation $A\vec{x} = \vec{0}$ has non-trivial solutions so $T$ cannot be one-to-one.**

*Date*: February 1, 2013.