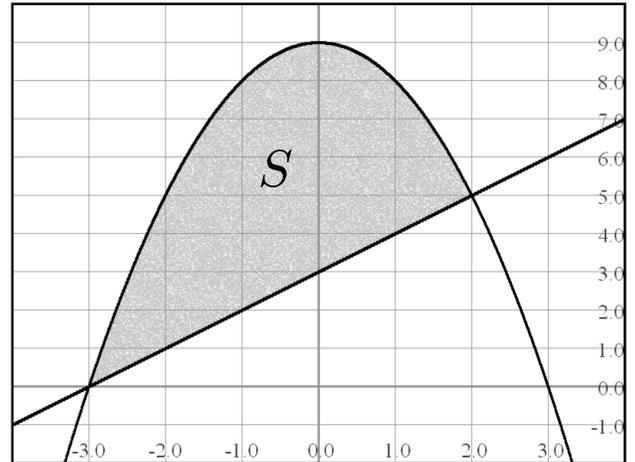


1. Consider the region S in the plane of all points which are between the graphs of the parabola $y = 9 - x^2$ and the straight line $y = x + 3$. The region S is shown in the figure to the right.

1A. Set up the integral which represents the area of S if the corresponding approximations are rectangles each of whose base width is Δx and each rectangle goes from a top curve down to a bottom curve. That is, the integral is of the form $\int dx$.



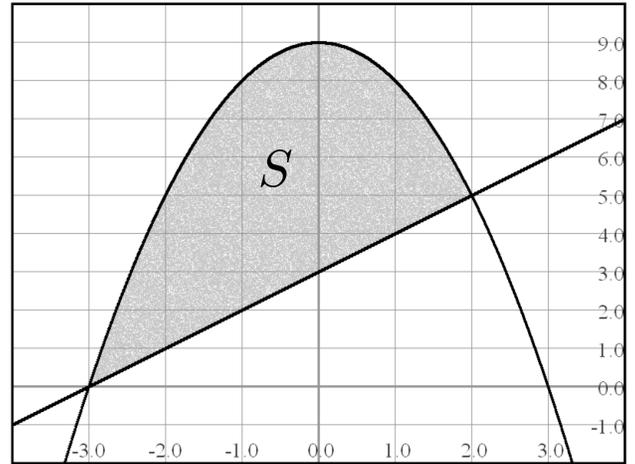
1B. Evaluate the integral in (1A) to find that area. Show all your work.

2A. Set up the integral which gives the arc length of the curved part of the boundary of S , that is, the arc length of the graph of $y = 9 - x^2$ from $x = -3$ to $x = 2$.

2B. The integral in 2A is “doable” — the back of your book has a formula for the antiderivative you’d need. But it’s so complicated, and in practice, a good numerical approximation will do. Indeed, find the MID(50) approximation for the integral in 2A.

3. Again, consider the region S from problem 1, of all points which are between the graphs of the parabola $y = 9 - x^2$ and the straight line $y = x + 3$. The region S is shown in the figure to the right.

3A. Set up the integral(s) which represents the volume of the solid of revolution obtained by revolving S around the line $y = -1$. (Do not evaluate the integral(s)).



3B. Now set up the integral(s) which represents the volume of the solid of revolution obtained by revolving S around the line $x = -3$. (Do not evaluate the integral(s)).

4. Let I be the exact value of $\int_0^2 (\sin(x) + \cos(2x)) dx$ (you do not have to find I).

4A: Use an appropriate graph on your calculator to estimate the value of K_1 you should use in “theorem 3” to find the maximum possible error if a LHS is used as an approximation for I . (Give K_1 to **two** places after the decimal point).

$$K_1 = \boxed{}$$

4B: For the correct value of K_1 in (4A), find that maximum error guaranteed by theorem 3, if LHS(10) is used to approximate I . (Answer using **five** digits after the decimal point). Show the formula you use in your work.

4C: Use an appropriate graph to find the value of K_2 you should use in theorem 3 to find an error bound for either a TRAP or MID approximation of I . **Fact:** $(\sin(x) + \cos(2x))'' = -\sin(x) - 4\cos(2x)$

$$K_2 = \boxed{}$$

4D: Use the value of K_2 obtained in (4C) to find the smallest value of n you can choose so that theorem 3 guarantees MID(n) will be within 0.001 of I . *Show all your computations.*

4E: Find MID(n) for the value of n from (4D) to **seven** digits after the decimal point.

4F: Fact: $I = 1 - \cos(2) + (1/2)\sin(4)$. What is (to seven places after the decimal point) the actual error in using the answer to (4E)? (That is, find $|I - \text{MID}(n)|$ for the right value of n).

5. The following table of velocities $v(t)$ in feet per second at various times t for some moving object was recorded during an experiment:

t	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$v(t)$	-27	-21.1	-15.4	-10.3	-5.8	-2.0	1.3	4.0	6.4	8.4	10.1

5A. What is the physical meaning of $\int_1^4 v(t) dt$?

5B. For the integral $\int_1^4 v(t) dt$ in (5A), estimate LHS(n), RHS(n), TRAP(n) and MID(n) for the maximum possible number of subintervals n in each case, using only the information available in the table. Clearly label all your answers!

5C. The table suggests that $v(t)$ is an increasing, concave-down function on $[0, 5]$. Suppose it is, and there was enough information to find $I = \int_0^5 v(t) dt$ for each of LHS(n), RHS(n), TRAP(n) and MID(n) with $n = 25$. From smallest to largest, put these numbers in order: I , LHS(n), RHS(n), TRAP(n) and MID(n).

6A. Find $\int_1^4 \frac{\cos(8\sqrt{x})}{\sqrt{x}} dx$ by the method of substitution, using appropriate notation throughout.

6B. In particular, what are the limits on the integral in (6A) after the appropriate substitution is made?