

Math 106 Winter 2013

Test 1 (50 points)

Name: _____

Show all your work to receive full credit for a problem.

Do not use the calculator integral function. Whenever possible, find the exact values of integrals by finding antiderivatives or using the table of integrals.

When you use a formula from the table of integrals, mention the formula number and the value(s) of any constant(s) that you may need.

Round off your answers to four decimal places.

Include units in your answers wherever possible.

There are seven questions. Questions are printed on both sides of a page.

You may use any of the following facts:

$$\text{Arclength} = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

$$|I - L_n| \leq \frac{K_1(b-a)^2}{2n}$$

$$|I - R_n| \leq \frac{K_1(b-a)^2}{2n}$$

$$|I - T_n| \leq \frac{K_2(b-a)^3}{12n^2}$$

$$|I - M_n| \leq \frac{K_2(b-a)^3}{24n^2}$$

Below are product rule, quotient rule and chain rule for derivatives.

$$(uv)' = u'v + uv'$$

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

$$(f \circ g)'(x) = f'(g(x))g'(x)$$

1. (8 points) Evaluate the following integral exactly. (You may use formulas 1-18 only from the table of integrals for this problem.)

$$\int_0^{\pi/6} \frac{\cos(3x)}{4 + \sin^2(3x)} dx$$

2. (6 points) Write (but do not evaluate) an integral to find the area of the region bounded by the curves $y = x$, $y = \sqrt{x}$, $x = 0$ and $x = 2$.

3. (8 points) An ant is walking along the curve $y = 3x^2$. Find the exact distance traveled by the ant as it walks from the point $(0, 0)$ to the point $(1, 3)$ along the curve.

4. (6 points) Let $I = \int_{0.5}^2 x \ln x \, dx$. What is the least value of n which guarantees that R_n approximates I within ± 0.01 ? Justify your answer.

5. (6 points) Let $I = \int_1^3 e^{-f(x)} \, dx$ where $f(x)$ is a function with the following properties:

$6 \leq f'(x) \leq 8$ and $-4 \leq f''(x) \leq 0$ for all x in the interval $[1, 3]$.

(a) Does L_{50} overestimate or underestimate I ? Justify your answer.

(b) Does M_{50} overestimate or underestimate I ? Justify your answer.

6. (8 points) Solve the IVP: $y' = \frac{1}{y^2 - 4y^2x}$, $y(0) = 2$. (You may use formulas 1-18 only from the table of integrals for this problem.)

7. (8 points)

- (a) Sketch the region bounded by the curve $y = \ln x$, and the lines $x = 2$ and $y = 0$. Write (but do not evaluate) an integral to find the volume of the solid that is formed when the region is rotated about the y -axis.

- (b) Write (but do not evaluate) an integral to find the volume of the hole in the solid in part(a).