**MATH106A,B CALCULUS II - PROF. P. WONG**

EXAM I - JANUARY 31, 2014

**NAME:**

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

*Advice*: DON'T spend too much time on a single problem.

| **Problems** | **Maximum Score** | **Your Score** |
|---|---|---|
| 1. | 20 | |
| 2. | 20 | |
| 3. | 20 | |
| 4. | 20 | |
| 5. | 20 | |
| **Total** | 100 | |

**1.**(10 pts.)(a) Evaluate the indefinite integral (be sure to show all your work)

$$\int x^2(x^3+6)^8 \, dx.$$

(10 pts.) (b) Find the **exact value** of the definite integral (be sure to show all your work)

$$\int_0^4 \sqrt{x^2+x}(2x+1) \, dx.$$

**2.** Consider the region $A$ bounded by the curve $y = x^3 - x^2 - 2x + 5$ and the line $y = 5$.

(15 pts.) Find the **exact area** of the region $A$.

(5 pts.) Let $B$ be the region bounded by the same two curves and the line $x = 2$ that lies in the first quadrant. Write a definite integral (do not evaluate) representing the area of region $B$. [By area, we mean the usual *physical* area not *signed* area.]

**3.** (12 pts.) Consider a function $g$ on the interval $[0, 2]$.

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|
| $g(x)$ | -1 | 1 | 3 | 2 | -1 | -3 | -2 |

Find $R_6, M_3$ using the right-hand sum and the mid-point rule respectively for estimating the definite integral $\int_0^3 g(x)\,dx$.

(8 pts.)(b) Recall that the error committed by using the left hand sum approximation $L_n$ is less than or equal to $\frac{K_1 \cdot (b-a)^2}{2n}$ where $|f'(x)| \leq K_1$ for some constant $K_1$ over the interval $[a, b]$. Use this result to give an upper bound for the error committed by $L_{10}$ for

$$I = \int_4^6 e^{-x} \sin x \; dx.$$

**4.** Let $R$ be the region bounded by the curve $x = 2\sqrt{y}$, the line $y = 4$, and the line $x = 0$.

(12 pts.) (a) Find the **exact volume** of the solid obtained from rotating the region $R$ around the $y$-axis. [Hint: sketch a picture of the region $R$ first.]

(8 pts.) (b) Set up (do not evaluate) a definite integral representing the volume of the solid obtained from rotating the region $R$ around the $x$-axis.

**5.** Consider the initial value problem

$$\frac{dy}{dx} = \frac{x}{y}$$

with $y(1) = 1$.

(10 pts.)(a) Use the technique of separation of variables to solve the Initial Value Problem.

(10 pts.)(b) Find the arc length of the portion of the graph of $f(x) = 4x^{3/2}$ between $x = 0$ and $x = 1$.