

Math 106 Winter 2014

Test 1 (50 points)

Name: _____

Show all your work to receive full credit for a problem. Points will be taken off if you do not show how you arrived at your answer, even if the final answer is correct.

Please keep your written answers brief; be clear and to the point. Points will be taken off for rambling and for incorrect or irrelevant statements and for multiple solutions to the same problem.

Do not use the calculator integral function. Whenever possible, find the exact values of integrals by finding antiderivatives or using the table of integrals.

When you use a formula from the table of integrals, mention the formula number and the value(s) of any constant(s) that you may need.

Give exact answers. If needed, round off your answers to four decimal places.

Include units in your answers wherever possible.

There are six questions. Questions are printed on both sides of a page.

You may use any of the following facts:

$$\text{Arclength} = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

$$|I - L_n| \leq \frac{K_1(b-a)^2}{2n}$$

$$|I - R_n| \leq \frac{K_1(b-a)^2}{2n}$$

$$|I - T_n| \leq \frac{K_2(b-a)^3}{12n^2}$$

$$|I - M_n| \leq \frac{K_2(b-a)^3}{24n^2}$$

Below are product rule, quotient rule and chain rule for derivatives.

$$(uv)' = u'v + uv'$$

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

$$(f \circ g)'(x) = f'(g(x))g'(x)$$

1. (8 points) Evaluate the following integral exactly. (You may use formulas 1-18 only from the table of integrals for this problem.)

$$\int_4^9 \frac{7e^{\sqrt{x}} - 1}{\sqrt{x}} dx .$$

2. (8 points) Solve the DE: $y' \sin^2(2x) = y^2 \cos(2x)$. (You may use formulas 1-18 only from the table of integrals for this problem.)

3. (7 points) Sketch the region bounded by the curve $y = \ln x$, the line $y = 1$, the x -axis and the y -axis. Write (but do not evaluate) an integral to find the volume of the solid obtained by rotating the region about the x -axis.

4. (8 points) Let $I = \int_2^6 f(x) dx$ where $f(x)$ is a function with the following properties:

$0 \leq f'(x) \leq 10$ and $-3 \leq f''(x) \leq 1$ for all x in the interval $[2, 6]$.

What is the least value of n which guarantees that T_n approximates I within ± 0.05 ? Justify your answer.

5. (7 points) Find the exact length of the curve $y = 4 - 5x^2$ from $x = 0$ to $x = 1$.

6. (12 points) Water flows into a storage tank at a rate of $r(t)$ ft³/min, where t is the number of minutes since the water starts to flow in. The table below gives data for t and $r(t)$. Assume that $r(t)$ is increasing on the interval $[0, 7]$. Let $I = \int_1^7 r(t) dt$.

t	1	2	3	4	5	6	7
$r(t)$	30	33	34	36	37	39	42

- (a) What does the integral $\int_1^7 r(t) dt$ represent?
- (b) Use the table to find the approximation L_3 to I . Does L_3 underestimate or overestimate I ? Justify your answer.
- (c) Indicate whether the following statement must be true, cannot be true or may be true. Justify your answer.

$$M_{20} < I.$$