Math 205 A & B Quiz 02 ver02 page 1 01/30/2015 **Name** suggested solutions

1. Let $A = \begin{bmatrix} 2 & -4 & 1 & 8 \\ 5 & -10 & -1 & 13 \\ -3 & 6 & 1 & -7 \end{bmatrix}$. 1A) Let $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$. Use the "super-augmented" matrix technique developed in class to find any/all conditions on u_1, u_2, and u_3 that guarantee the matrix equation $A\mathbf{x} = \mathbf{u}$ is consistent. Show both the super-augmented matrix you use and its RREF.

Consider $\left[\begin{array}{cccc|ccc} 2 & -4 & 1 & 8 & 1 & 0 & 0 \\ 5 & -10 & -1 & 13 & 0 & 1 & 0 \\ -3 & 6 & 1 & -7 & 0 & 0 & 1 \end{array}\right]$;

its RREF is $\left[\begin{array}{cccc|ccc} 1 & -2 & 0 & 3 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1 & 2 & 0 & 3/2 & 5/2 \\ 0 & 0 & 0 & 0 & 1 & -5/2 & -7/2 \end{array}\right]$

The last row requires $\boxed{0 = u_1 - \tfrac{5}{2}u_2 - \tfrac{7}{2}u_3}$ or $\boxed{u_1 = \tfrac{5}{2}u_2 + \tfrac{7}{2}u_3}$ in order for $A\vec{x} = \vec{u}$ to be consistent

1B) Show that the vector $\mathbf{v} = \begin{bmatrix} 1 \\ -8 \\ 6 \end{bmatrix}$ satisfies the condition(s) you found in (1A)

Does $1 \stackrel{?}{=} \tfrac{5}{2}(-8) + \tfrac{7}{2}(6)$
$= 5(-4) + 7\cdot 3$
$= -20 + 21$? yes

1C) Find all the solutions of $A\mathbf{x} = \mathbf{v}$ and express them in parametric vector form.

$[A|\vec{v}] = \left[\begin{array}{cccc|c} 2 & -4 & 1 & 8 & 1 \\ 5 & -10 & -1 & 13 & -8 \\ -3 & 6 & 1 & -7 & 6 \end{array}\right]$

has RREF $\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -1 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ which tells us the solutions $\vec{x}$ are $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 + 2x_2 - 3x_4 \\ x_2 \\ 3 - 2x_4 \\ x_4 \end{bmatrix}$ where x_2 & x_4 are free.

$$\boxed{= \begin{bmatrix} -1 \\ 0 \\ 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ 0 \\ -2 \\ 1 \end{bmatrix}}$$ "

1D) What specific solution $\mathbf{s} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$ do you get for $A\mathbf{x} = \mathbf{v}$ if you set all the free variables in (1C) to 10?

we find $\vec{s} = \begin{bmatrix} -1 + 20 - 30 \\ 10 \\ 3 - 20 \\ 10 \end{bmatrix} = \begin{bmatrix} -11 \\ 10 \\ -17 \\ 10 \end{bmatrix}$

1E) Find $A\mathbf{s}$ (do this product using your calculator and two of its matrices as shown in class) for the vector $\mathbf{s}$ in (1D). What do you get for $A\mathbf{s}$?

Since $\vec{s}$ is a solution of $A\vec{x} = \vec{v}$, we get $A\vec{s} = \begin{bmatrix} 1 \\ -8 \\ 6 \end{bmatrix}$!!! (no need to use a calculator unless you'd like to verify this!)