1. Let $A = \begin{bmatrix} 3 & -6 & 6 & 24 \\ -5 & 10 & 3 & -1 \\ 7 & -14 & 1 & 17 \end{bmatrix}$. 1A) Let $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$. Use the "super-augmented" matrix technique developed in class to find any/all conditions on u_1, u_2, and u_3 that guarantee the matrix equation $A\mathbf{x} = \mathbf{u}$ is consistent. Show both the super-augmented matrix you use and its RREF.

Consider $\left[\begin{array}{cccc|ccc} 3 & -6 & 6 & 24 & 1 & 0 & 0 \\ -5 & 10 & 3 & -1 & 0 & 1 & 0 \\ 7 & -14 & 1 & 17 & 0 & 0 & 1 \end{array}\right]$;

its RREF is

$$\left[\begin{array}{cccc|ccc} 1 & -2 & 0 & 2 & 0 & -1/26 & 3/26 \\ 0 & 0 & 1 & 3 & 0 & 7/26 & 5/26 \\ 0 & 0 & 0 & 0 & 1 & -3/2 & -3/2 \end{array}\right]$$

so $A\vec{x} = \vec{u}$ is consistent $\Leftrightarrow$ $\boxed{0 = u_1 - \frac{3}{2}u_2 - \frac{3}{2}u_3}$

or, $\boxed{u_1 = \frac{3}{2}u_2 + \frac{3}{2}u_3}$

1B) Show that the vector $\mathbf{v} = \begin{bmatrix} 21 \\ 30 \\ -16 \end{bmatrix}$ satisfies the condition(s) you found in (1A)

Does $21 \stackrel{?}{=} \frac{3}{2}(30) + \frac{3}{2}(-16)$

$= 45 - 24$? yes!

1C) Find all the solutions of $A\mathbf{x} = \mathbf{v}$ and express them in parametric vector form.

$$\left[\begin{array}{cccc|c} 3 & -6 & 6 & 24 & 21 \\ -5 & 10 & 3 & -1 & 30 \\ 7 & -14 & 1 & 17 & -16 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -2 & 0 & 2 & -3 \\ 0 & 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow$$

the solutions are $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3 + 2x_2 - 2x_4 \\ x_2 \\ 5 - 3x_4 \\ x_4 \end{bmatrix}$ where x_2 and x_4 are free.

$$\boxed{= \begin{bmatrix} -3 \\ 0 \\ 5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 0 \\ -3 \\ 1 \end{bmatrix}}$$

1D) What specific solution $\mathbf{s} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$ do you get for $A\mathbf{x} = \mathbf{v}$ if you set all the free variables in (1C) to 10?

We find $\vec{s} = \begin{bmatrix} -3 + 20 - 20 \\ 10 \\ 5 - 30 \\ 10 \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \\ -25 \\ 10 \end{bmatrix}$

1E) Find $A\mathbf{s}$ (do this product using your calculator and two of its matrices as shown in class) for the vector $\mathbf{s}$ in (1D). What do you get for $A\mathbf{s}$?

Since $\vec{s}$ is a solution of $A\vec{x} = \vec{v}$, we should find $A\vec{s}$ is $\begin{bmatrix} 21 \\ 30 \\ -16 \end{bmatrix}$, (and there's no need for the calculator unless you want to verify the result!)