MATH 205A,B - LINEAR ALGEBRA
WINTER 2013

QUIZ 3

NAME: **Section:**(Circle one) A(1 : 10) B(2 : 40)

Show ALL your work CAREFULLY.

Let

$$A = \begin{bmatrix} 1 & -2 & -1 \\ -1 & 2 & 2 \\ 2 & -4 & -1 \end{bmatrix}.$$

(a) Express the solutions to the homogeneous equation $A\vec{x} = \vec{0}$ in parametric form.

The coefficient matrix A can be reduced as follows.

$$\begin{bmatrix} 1 & -2 & -1 \\ -1 & 2 & 2 \\ 2 & -4 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 \\ 0 & 0 & 1 \\ 2 & -4 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

The solutions to $A\vec{x} = \vec{0}$ are

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_2 \\ x_2 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

where x_2 is the parameter.

(b) Based on your answer to (a), determine whether the columns of A are linearly independent? Justify your answer.

The columns of A are NOT linearly independent since $\vec{0}$ can be written as a non-trivial linear combinations of the columns $\vec{a}_1, \vec{a}_2, \vec{a}_3$. For instance, $\vec{0} = (2)\vec{a}_1 + (1)\vec{a}_2 + (0)\vec{a}_3$ (take $x_2 = 1$).

(c) Do the columns of A span $\mathbb{R}^3$? Explain.

The columns of A do NOT span $\mathbb{R}^3$. For any vector $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ where $b_3 \neq 0$, the system $A\vec{x} = \vec{b}$ does not have a solution because the row reduced echelon form of A has a row of zeroes and thus the system would be inconsistent.

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