Math 205 A & B Quiz 02 ver01 page 1 01/24/2014 Name suggested soln

1. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ and $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

1A) Find a vector $\mathbf{b} \in \mathbb{R}^3$ different from both $\mathbf{a}_1$ and $\mathbf{a}_2$ that is in Span$\{\mathbf{a}_1, \mathbf{a}_2\}$ and explain how you found $\mathbf{b}$.

The span is made up of all Linear Combinations $x_1\vec{a}_1 + x_2\vec{a}_2$, so we'll just pick a couple values for the weights x_1 & x_2 to create $\vec{b}$. For example, let $x_1 = x_2 = 1$ to get

$$\vec{b} = 1\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$$

even easier, let $x_1 = x_2 = 0$ to get $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \vec{0}$.

1B) Find a vector $\mathbf{d} \in \mathbb{R}^3$ that is obviously *not* in Span$\{\mathbf{a}_1, \mathbf{a}_2\}$ and explain why $\mathbf{d}$ is not in that span.

The "middle" entry of $x_1\vec{a}_1 + x_2\vec{a}_2$ is $x_1 0 + x_2 0$, which is always 0. So if $\vec{d} = \begin{bmatrix} 0 \\ c \\ 0 \end{bmatrix}$ where c is a non-zero number, then $\vec{d}$ is NOT in span$\{\vec{a}_1, \vec{a}_2\}$. For example, let $\vec{d} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

2. Let $\mathbf{c}_1 = \begin{bmatrix} 4 \\ 2 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{c}_2 = \begin{bmatrix} 8 \\ 4 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{c}_3 = \begin{bmatrix} 5 \\ 5 \\ 2 \\ 2 \end{bmatrix}$, and $\mathbf{c}_4 = \begin{bmatrix} -1 \\ 7 \\ 6 \\ 2 \end{bmatrix}$. Also, let $\mathbf{v} = \begin{bmatrix} 22 \\ 16 \\ 4 \\ 7 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 8 \\ 11 \\ 7 \\ 4 \end{bmatrix}$.

2A: Determine if $\mathbf{v}$ a linear combination of $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$, and $\mathbf{c}_4$, and if it is, find all possible solutions of the vector equation $x_1\mathbf{c}_1 + \cdots + x_4\mathbf{c}_4 = \mathbf{v}$. Express your values of the x_i's in the agreed-upon format. Show any RREF matrices used in your solution.

The RREF of $\left[\begin{array}{cccc|c} 4 & 8 & 5 & -1 & 22 \\ 2 & 4 & 5 & 7 & 16 \\ 0 & 0 & 2 & 6 & 4 \\ 1 & 2 & 2 & 2 & 7 \end{array}\right]$ is needed to answer this question.

But that RREF is $\left[\begin{array}{cccc|c} 1 & 2 & 0 & -4 & 3 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$, and it tells us the vector equation $x_1\vec{c}_1 + x_2\vec{c}_2 + x_3\vec{c}_3 + x_4\vec{c}_4 = \vec{v}$ is consistent, and indeed there are ∞-many solns:

$$\begin{cases} x_1 = 3 - 2x_2 + 4x_3 \\ x_2 \text{ is free} \\ x_3 = 2 - 3x_3 \\ x_4 \text{ is free} \end{cases}$$

[just FYI: from row 1, from row 3, b/c x_2 & x_4 correspond to non-pivot columns]

Therefore, $\vec{v}$ is a L.C. of $\vec{c}_1, \vec{c}_2, \vec{c}_3$ and $\vec{c}_4$.

2B: Explain whether $\mathbf{w}$ is in the Span of $\{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \mathbf{c}_4\}$. Again show any RREF matrices used in your explanation.

here the RREF of the augmented matrix corresponding to the vector eq'n

$$x_1\vec{c}_1 + x_2\vec{c}_2 + x_3\vec{c}_3 + x_4\vec{c}_4 = \vec{w} \quad (*)$$

is $\left[\begin{array}{cccc|c} 1 & 2 & 0 & -4 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ ← this row represents the eqn $0x_1 + 0x_2 + 0x_3 + 0x_4 = 1$, which has no soln's, so neither does eqn $(*)$

Therefore, $\vec{w}$ is NOT in the span$\{\vec{c}_1, \vec{c}_2, \vec{c}_3, \vec{c}_4\}$

[FYI: "the matrix is inconsistent" is INCORRECT. SAY "the system represented by the matrix is inconsistent".]