Math 205 A & B Quiz 01 $\boxed{\text{ver02}}$ page 1 $\boxed{\text{Q1}}$ 01/23/2015 **Name** Suggested solutions

1. Consider the following vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 5 \\ 6 \\ 3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 5 \\ 12 \\ 9 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 5 \\ 10 \\ 7 \end{bmatrix} \text{ and } \mathbf{b}_1 = \begin{bmatrix} 5 \\ 14 \\ 11 \end{bmatrix}.$$

1a. Does the vector equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 + x_4\mathbf{v}_4 = \mathbf{b}_1$ have any solutions (x_1, x_2, x_3, x_4)? If so, write the solutions in our standard notation with all "basic" variables in terms of the free variables (if there are any free variables). But if there are no solutions, explain why not. Show any augmented matrices and their RREF's that you use in answering this question.

the augmented matrix which represents the equation is

$$\left[\begin{array}{cccc|c} 5 & 5 & 5 & 5 & 5 \\ 6 & 4 & 12 & 10 & 14 \\ 3 & 1 & 9 & 7 & 11 \end{array}\right]. \quad \text{Its RREF is} \quad \left[\begin{array}{cccc|c} 1 & 0 & 4 & 3 & 5 \\ 0 & 1 & -3 & -2 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

which represents the equations
$$\begin{cases} x_1 \quad +4x_3+3x_4=5 \\ x_2-3x_3-2x_4=-4 \\ 0x_1+0x_2+0x_3+0x_4=0 \end{cases}$$
which we know has solns
$$\begin{cases} x_1 = 5-4x_3-3x_4 \\ x_2 = -4+3x_3+2x_4 \\ \text{where } x_3 \text{ and } x_4 \\ \text{are free.} \end{cases}$$

[note the last eqn is solved by ANY $x_1\ x_2\ x_3\ x_4$ and is therefore useless]

1b. If there are solutions in (1a), set all the free variables to 1 (if there are any free variables) and verify that the resulting solution does indeed work in the equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 + x_4\mathbf{x}_4 = \mathbf{b}_1$.

with $x_3 = 1$ and $x_4 = 1$ we find $x_1 = 5-4-3 = 5-7 = -2$ AND $x_2 = -4+3+2 = -4+5 = 1$

now lets check that the solution $(-2,1,1,1)$ "works":
$$-2\begin{bmatrix} 5 \\ 6 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix} + 1\begin{bmatrix} 5 \\ 12 \\ 9 \end{bmatrix} + 1\begin{bmatrix} 5 \\ 10 \\ 7 \end{bmatrix} = \begin{bmatrix} -10+5+5+5 \\ -12+4+12+10 \\ -6+1+9+7 \end{bmatrix} = \begin{bmatrix} -10+15 \\ -12+26 \\ -6+17 \end{bmatrix} \checkmark = \begin{bmatrix} 5 \\ 14 \\ 11 \end{bmatrix} \text{ as expected.}$$

2. In general, an expression such as $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_p\mathbf{v}_p$ is called a what?

a Linear Combination of the vectors $\vec{v}_1, \cdots, \vec{v}_p$.

3. Let $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$ be as in question 1. Let $\mathbf{b}_2 = \begin{bmatrix} 5 \\ 7 \\ 8 \end{bmatrix}$. Is $\mathbf{b}_2$ in Span($\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$)? Fully explain your answer, and again show any augmented matrices and their RREF's that you use.

now the corresponding augmented matrix is

$$\left[\begin{array}{cccc|c} 5 & 5 & 5 & 5 & 5 \\ 6 & 4 & 12 & 10 & 7 \\ 3 & 1 & 9 & 7 & 8 \end{array}\right], \text{ and its RREF is } \left[\begin{array}{cccc|c} 1 & 0 & 4 & 3 & 0 \\ 0 & 1 & -3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

The last row of this RREF represents the equation $0x_1 + 0x_2 + 0x_3 + 0x_4 = 1$, which has no solutions for any x_1, x_2, x_3, x_4. Thus the eqn $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 + x_4\vec{v}_4 = \vec{b}_2$ also has no solutions, and so NO, $\vec{b}_2$ is not in the span of the vectors $\vec{v}_1, \cdots, \vec{v}_4$.