1. Consider the following vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 6 \\ 4 \\ -1 \end{bmatrix} \text{ and } \mathbf{b}_1 = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}.$$

1a. Does the vector equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 + x_4\mathbf{v}_4 = \mathbf{b}_1$ have any solutions (x_1, x_2, x_3, x_4)? If so, write the solutions in our standard notation with all "basic" variables in terms of the free variables (if there are any free variables). But if there are no solutions, explain why not. Show any augmented matrices and their RREF's that you use in answering this question.

1b. If there are solutions in (1a), set all the free variables to 1 (if there are any free variables) and verify that the resulting solution does indeed work in the equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 + x_4\mathbf{x}_4 = \mathbf{b}_1$.

2. In general, an expression such as $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_p\mathbf{v}_p$ is called a what?

3. Let $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$ be as in question 1. Let $\mathbf{b}_2 = \begin{bmatrix} 8 \\ 4 \\ -2 \end{bmatrix}$. Is $\mathbf{b}_2$ in Span($\{\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3,\ \mathbf{v}_4\}$)? Fully explain your answer, and again show any augmented matrices and their RREF's that you use.