MATH 205A,B - LINEAR ALGEBRA
WINTER 2013

QUIZ 2

NAME: **Section:**(Circle one) A(1 : 10) B(2 : 40)

Show ALL your work CAREFULLY.

Let

$$A = \begin{bmatrix} 0 & 0 & 4 \\ 0 & -3 & -1 \\ -2 & 8 & -5 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 4 \\ -7 \\ 13 \end{bmatrix}.$$

(a) Write the vector equation corresponding to the matrix equation $A\vec{x} = \vec{b}$.

Since the matrix A has 3 rows, the solutions of the corresponding system have 3 variables, x_1, x_2 and x_3. The following is the corresponding vector equation:

$$x_1 \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \\ 13 \end{bmatrix}.$$

(b) Does the matrix equation $A\vec{x} = \vec{b}$ have a solution? If yes, find one.

The corresponding augmented matrix is

$$\begin{bmatrix} 0 & 0 & 4 & 4 \\ 0 & -3 & -1 & -7 \\ -2 & 8 & -5 & 13 \end{bmatrix}.$$

Using elementary row operations, the reduced row echelon form of A can be obtained as follows.

$$\begin{bmatrix} 0 & 0 & 4 & 4 \\ 0 & -3 & -1 & -7 \\ -2 & 8 & -5 & 13 \end{bmatrix} \sim \begin{bmatrix} -2 & 8 & -5 & 13 \\ 0 & -3 & -1 & -7 \\ 0 & 0 & 4 & 4 \end{bmatrix} \sim \begin{bmatrix} -2 & 8 & -5 & 13 \\ 0 & -3 & -1 & -7 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} -2 & 8 & -5 & 13 \\ 0 & -3 & 0 & -6 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} -2 & 8 & -5 & 13 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} -2 & 8 & 0 & 18 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} -2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

THE solution is $x_1 = -1, x_2 = 2$ and $x_3 = 1$.

(c) Do the columns of A span $\mathbb{R}^3$? Explain.

YES, they do. From part (b), the reduced row echelon form of A is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

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It follows that for any arbitrary vector $\vec{b}$, the equation $A\vec{x} = \vec{b}$ always has a (unique) solution. Thus, every $\vec{b}$ is a linear combination of the columns of A.