1. Let M be the augmented matrix $\left[\begin{array}{cccc|c} 4 & 5 & -2 & 10 & 20 \\ 3 & 7 & 5 & 10 & 7 \\ 0 & 2 & 4 & 3 & -2 \\ 1 & 2 & 1 & 3 & 3 \end{array}\right]$.

1A: Let S be the system of equations having augmented matrix M. What is the second equation of that system?

$3x_1 + 7x_2 + 5x_3 + 10x_4 = 7$

1B: You might *begin* to put the matrix M in RREF by first performing the elementary row operation "$r_1 \leftrightarrow r_4$" on M to produce a row equivalent matrix N, and then doing the elementary row operation "$r_2 \leftarrow r_2 - 3r_1$" on N to get another row-equivalent matrix P. Explicitly, what is P?

So N is $\left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 3 \\ 3 & 7 & 5 & 10 & 7 \\ 0 & 2 & 4 & 3 & -2 \\ 4 & 5 & -2 & 10 & 20 \end{array}\right]$; P becomes $\left[\begin{array}{cccc|c} 1 & 2 & 1 & 3 & 3 \\ 0 & 1 & 2 & 1 & -2 \\ 0 & 2 & 4 & 3 & -2 \\ 4 & 5 & -2 & 10 & 20 \end{array}\right]$

1C: By calculator, what is RREF(M)?

$\text{RREF}(M) = \left[\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 5 \\ 0 & 1 & 2 & 0 & -4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

1D: What are the pivot columns of M? columns 1, 2, and 4 as these have the "leading 1's"

1E: Find all the solutions of the system S and write your answer in the agreed-upon form. If there are no solutions, explain why not.

$$\begin{cases} x_1 = 5 + 3x_3 \\ x_2 = -4 - 2x_3 \\ x_3 \text{ is free} \\ x_4 = 2 \end{cases}$$

1F: If the system S is not inconsistent, and there are any free variables, what solution results by setting all the free variables to 1? (Write your answer in "n-tuple" form).

Setting $x_3 = 1$ gives the soln $(5+3, -4-2, 1, 2) = (8, -6, 1, 2)$

1G: If there is an answer to 1F, verify that the solution is indeed a solution by seeing that it satisfies the equation in the answer to 1A.

Does $3(8) + 7(-6) + 5(1) + 10(2) = 7$?

$24 - 42 + 5 + 20 = 7$?

$49 - 42 = 7$?

$7 = 7$? yes.

(note: this only shows that $(8, -6, 1, 2)$ is a soln of the second eqn... not the whole system!)